Answer to Question #87341 - Math - Calculus

$$y = f(x) = \frac{2x^5 + x^2 - 5}{x^2}$$

$$f(x) = 2x^3 + 1 - 5x^{-2}$$

$$f(x) = 2x^3 - \frac{5}{x^2} + 1$$

Using First principle : $f'(x) = lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$,we get,

$$f'(x) = lim_{h\to 0} \frac{\{2(x+h)^3 - \frac{5}{(x+h)^2} + 1\} - \left\{2x^3 - \frac{5}{x^2} + 1\right\}}{h}$$

$$f'(x) = lim_{h\to 0} \frac{\left\{2\left(x^3 + h^3 + 3x^2h + 3xh^2\right) - \frac{5}{(x+h)^2} + 1\right\} - \left\{2x^3 - \frac{5}{x^2} + 1\right\}}{h}$$

$$f'(x) = lim_{h\to 0} \frac{2x^3 + 2h^3 + 6x^2h + 6xh^2 - \frac{5}{(x+h)^2} + 1 - 2x^3 + \frac{5}{x^2} - 1}{h}$$

$$f'(x) = lim_{h\to 0} \frac{2h^3 + 6x^2h + 6xh^2 - \frac{5}{(x+h)^2} + \frac{5}{x^2}}{h}$$

$$f'(x) = lim_{h\to 0} \frac{2h^3 + 6x^2h + 6xh^2 + \dfrac{-5x^2 + 5(x+h)^2}{x^2(x+h)^2}}{h}$$

$$f'(x) = lim_{h\to 0} \frac{2h^3 + 6x^2h + 6xh^2 + \dfrac{-5x^2 + 5x^2 + 5h^2 + 10xh}{x^2(x+h)^2}}{h}$$

$$f'(x) = lim_{h\to 0} \frac{2h^3 + 6x^2h + 6xh^2 + \dfrac{5h^2 + 10xh}{x^2(x+h)^2}}{h}$$

$$f'(x) = lim_{h\to 0} \frac{h\left\{2h^2 + 6x^2 + 6xh + \frac{5h+10x}{x^2(x+h)^2}\right\}}{h}$$

$$f'(x) = lim_{h\to 0} \left\{2h^2 + 6x^2 + 6xh + \frac{5h+10x}{x^2(x+h)^2}\right\}$$

Plugging limit $h = 0$

$$f'(x) = 2(0)^2 + 6x^2 + 6x(0) + \frac{5(0)+10x}{x^2(x+0)^2}$$

$$f'(x) = 6x^2 + \frac{10x}{x^2(x)^2}$$

$$f'(x) = 6x^2 + \frac{10}{x^3}.$$