Answer to Question #87286 - Math - Calculus

Question:

6. Evaluate the limit $\lim\limits_{x\to-\infty}\dfrac{2x^4 - x^2 + 8x}{-5x^4 + 7}$.

a. $\frac{1}{3}$ b. $\frac{2}{3}$ c. $\frac{1}{2}$ d. $\frac{3}{4}$

7. Evaluate the limit $\lim\limits_{t\to4}\dfrac{t-\sqrt{3t+4}}{4-t}$.

a. $-\frac{3}{8}$ b. $-\frac{5}{8}$ c. $-\frac{1}{8}$ d. $\frac{3}{4}$

Solution:

6. $\lim\limits_{x\to-\infty}\dfrac{2x^4 - x^2 + 8x}{-5x^4 + 7} = \lim\limits_{x\to-\infty}\dfrac{2\frac{x^4}{x^4} - \frac{x^2}{x^4} + 8\frac{x}{x^4}}{-5\frac{x^4}{x^4} + \frac{7}{x^4}} = \lim\limits_{x\to-\infty}\dfrac{2 - \frac{1}{x^2} + \frac{8}{x^3}}{-5 + \frac{7}{x^4}} = -\dfrac{2}{5}$ [Since $\lim\limits_{x\to-\infty}\dfrac{1}{x^n} = 0$].

7. $\lim\limits_{t\to4}\dfrac{t-\sqrt{3t+4}}{4-t} = \dfrac{4-\sqrt{3\times4+4}}{4-4} = \dfrac{0}{0}$ Indeterminate form.

Multiply by the conjugate,

$$\lim_{t\to4}\frac{t-\sqrt{3t+4}}{4-t}\times\frac{t+\sqrt{3t+4}}{t+\sqrt{3t+4}} = \lim_{t\to4}\frac{t^2-(3t+4)}{(4-t)\left(t+\sqrt{3t+4}\right)}$$

$$= \lim_{t\to4}\frac{t^2-3t-4}{(4-t)\left(t+\sqrt{3t+4}\right)} == \lim_{t\to4}\frac{(t-4)(t+1)}{(4-t)\left(t+\sqrt{3t+4}\right)}$$

$$= \lim_{t\to4}\frac{-(t+1)}{\left(t+\sqrt{3t+4}\right)} = \frac{-(4+1)}{4+\sqrt{3\times4+4}} = -\frac{5}{8}.$$