Answer to the Question #87111 – Math – Differential Equations

Question

Solve the differential equation d2y/dx2=a+bx+cx2 given that dy/dx=0 and y=d when x=0.

Solution

We want to solve the following differential equation:

$$\frac{d^2y}{dx^2} = a + bx + cx^2, \qquad \frac{dy}{dx}|_{x=0} = 0, \quad y(x=0) = d.$$

By integrating both sides of the equation and applying the Second Fundamental Theorem of Calculus we have

$$\int_0^t \frac{d^2y}{dx} dx = \int_0^t a + bx + cx^2 dx \Rightarrow$$

$$\frac{dy}{dx}|_{x=t} - \frac{dy}{dx}|_{x=0} = (ax + \frac{1}{2}bx^2 + \frac{1}{3}cx^3)|_{x=t} - (ax + \frac{1}{2}bx^2 + \frac{1}{3}cx^3)|_{x=0}$$

$$\Rightarrow \frac{dy}{dt} = at + \frac{1}{2}bt^2 + \frac{1}{3}ct^3 .$$

By doing the same process on the last equation we get

$$\int_0^s \frac{dy}{dt} dt = \int_0^s at + \frac{1}{2}bt^2 + \frac{1}{3}ct^3 dt \Rightarrow$$

$$y(s) - y(0) = (\frac{1}{2}at^2 + \frac{1}{6}bt^3 + \frac{1}{12}ct^4)|_{t=s} - (\frac{1}{2}at^2 + \frac{1}{6}bt^3 + \frac{1}{12}ct^4)|_{t=0}$$

$$\Rightarrow y(s) = \frac{1}{2}as^2 + \frac{1}{6}bs^3 + \frac{1}{12}cs^4 + d .$$

Please note that in the above calculations we have used the initial conditions given in the question. Thus, the solution of the differential equation is

$$y = \frac{1}{2}ax^2 + \frac{1}{6}bx^3 + \frac{1}{12}cx^4 + d .$$