Answer to the question #86822 – Math – Calculus

Question

Show that two scalar fields f and g

$\nabla .(\nabla f \times (f \nabla g)) = 0$

Solution

To prove this, we need the following identities:

$\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot \nabla \times \mathbf{A} - \mathbf{A} \cdot \nabla \times \mathbf{B}$ & $\nabla \times (f\mathbf{A}) = f\nabla \times \mathbf{A} + \nabla f \times \mathbf{A}$

(Please note that vectors are denoted by bold capital letters).

Thus, we have

$$\nabla \cdot (\nabla f \times (f\nabla g)) = (f\nabla g) \cdot (\nabla \times (\nabla f)) - (\nabla f) \cdot (\nabla \times (f\nabla g)) = -(\nabla f) \cdot (\nabla \times (f\nabla g))$$
$$= -(\nabla f) \cdot (f\nabla \times (\nabla g) + \nabla f \times \nabla g) = -\nabla f \cdot (\nabla f \times \nabla g) = 0$$

(Please note that in above we have used the following facts:

$\nabla \times (\nabla f) = 0$ & $\mathbf{A} \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{0}$).

For any scalar function f we have $\nabla \times (\nabla f) = 0$. So you have the same result for the scalar function g, i.e., $\nabla \times (\nabla g) = 0$. Now by multiplying the both sides of this equation by f we obtain $f \nabla \times (\nabla g) = f \times 0 = 0$.

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