

ANSWER TO QUESTION #86599 – MATH – STATISTICS AND PROBABILITY

QUESTION

The distribution of the binomial random variable (X) has the following parameters $p = 0.3$ and $n = 9$. Determine $E(X)$.

SOLUTION

Given $X \sim B(n, p)$

$$P(X = r) = \binom{n}{r} p^r (1 - p)^{n-r} \quad \text{where } r = 0, 1, 2, \dots, n;$$

$$E(X) = \sum_{r=0}^n r \binom{n}{r} p^r (1 - p)^{n-r}.$$

$$\text{Since } r \binom{n}{r} = \frac{r \cdot n!}{n-r! \cdot r!} = \frac{r \cdot n \cdot n-1!}{n-r! \cdot r \cdot r-1!} = \frac{n \cdot n-1!}{n-r! \cdot r-1!} = \frac{n \cdot n-1!}{(n-1)-(r-1)! \cdot r-1!} = n \binom{n-1}{r-1}$$

$$\begin{aligned} \text{Hence } E(X) &= \sum_{r=0}^n r \binom{n}{r} p^r (1 - p)^{n-r} = \sum_{r=1}^n np \binom{n-1}{r-1} p^{r-1} (1 - p)^{(n-1)-(r-1)} \\ &= np (p + 1 - p)^{n-1} = np \end{aligned}$$

Given $n = 9$, $p = 0.3$ hence $E(X) = np = 9(.3) = 2.7$.

Answer: $E(X) = 2.7$.