Answer to Question #86486 – Math – Algebra

Question

Apply Cramer's rule to solve the following system of equations:

$2x + y + z = 4$

$x - y + 2z = 2$

$3x - 2y - z = 0$

Solution

The system of equations is:

$2x + y + z = 4$

$x - y + 2z = 2$

$3x - 2y - z = 0$

The value of the coefficient determinant is:

$$D = \begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & 2 \\ 3 & -2 & -1 \end{vmatrix} = 2(1+4) - 1(-1-6) + 1(-2+3)$$

$$D = 2(5) - 1(-7) + 1(1) = 10 + 7 + 1 = 18$$

Now we have,

$$D_x = \begin{vmatrix} 4 & 1 & 1 \\ 2 & -1 & 2 \\ 0 & -2 & -1 \end{vmatrix} = 4(1+4) - 1(-2-0) + 1(-4+0)$$

$$D_x = 4(5) - 1(-2) + 1(-4) = 20 + 2 - 4 = 18$$

$$D_y = \begin{vmatrix} 2 & 4 & 1 \\ 1 & 2 & 2 \\ 3 & 0 & -1 \end{vmatrix} = 2(-2-0) - 4(-1-6) + 1(0-6)$$

$$D_y = 2(-2) - 4(-7) + 1(-6) = -4 + 28 - 6 = 18$$

$$D_z = \begin{vmatrix} 2 & 1 & 4 \\ 1 & -1 & 2 \\ 3 & -2 & 0 \end{vmatrix} = 2(0+4) - 1(0-6) + 4(-2+3)$$

$$D_z = 2(4) - 1(-6) + 4(1) = 8 + 6 + 4 = 18$$

Therefore using Cramer's rule we get,

$$x = \frac{D_x}{D} = \frac{18}{18} = 1$$

$$y = \frac{D_y}{D} = \frac{18}{18} = 1$$

$$z = \frac{D_z}{D} = \frac{18}{18} = 1$$

Hence, the solution to the system of equations is given by $x = 1, y = 1, z = 1$.

Answer: $x = 1, y = 1, z = 1$.