Answer on Question #86483 – Math – Calculus

Question

Find derivative of:

1) $\frac{\ln\left(x^2-3x+8\right)}{\sec\left(x^2+7x\right)}$
2) $\arctan\left(\cosh(2x-3)\right)$
3) $\left(\tan x\right)^{\ln x+x^2}$
4) $\left(\sec^2 x - \tan^2 x\right)^{45}$

Solution (1)

$$\left(\frac{\ln\left(x^2 - 3x + 8\right)}{\sec\left(x^2 + 7x\right)}\right)' =$$

$$= \frac{\left(\ln(x^2-3x+8)\right)' \cdot \sec(x^2+7x) - \ln(x^2-3x+8)\left(\sec(x^2+7x)\right)'}{\left(\sec(x^2+7x)\right)^2} =$$

$$= \frac{\ln'(x^2-3x+8) \cdot (x^2-3x+8)' \cdot \sec(x^2+7x) - \ln(x^2-3x+8)\sec'(x^2+7x)(x^2+7x)'}{\left(\sec(x^2+7x)\right)^2}$$

$$= \frac{\frac{1}{x^2-3x+8}(2x-3)\sec(x^2+7x) - \ln(x^2-3x+8)\sec(x^2+7x)\tan(x^2+7x)(2x+7)}{\left(\sec(x^2+7x)\right)^2}$$

$$= \frac{\sec(x^2+7x)\left(\frac{(2x-3)}{x^2-3x+8} - \ln(x^2-3x+8)\tan(x^2+7x)(2x+7)\right)}{\left(\sec(x^2+7x)\right)^2} =$$

$$= \frac{\left(\frac{(2x-3)}{x^2-3x+8} - \ln(x^2-3x+8)\tan(x^2+7x)(2x+7)\right)}{\sec(x^2+7x)} =$$

$$= \frac{(2x-3)}{(x^2-3x+8)\sec(x^2+7x)} - \frac{\ln(x^2-3x+8)\tan(x^2+7x)(2x+7)}{\sec(x^2+7x)}$$

Solution (2)

$$\left(\arctan\left(\cosh\left(2x-3\right)\right)\right)' = \arctan'\left(\cosh\left(2x-3\right)\right)\cdot\cosh'\left(2x-3\right)\left(2x-3\right)' =$$

$$= \frac{1}{1+\cosh^2(2x-3)}\cdot\sinh(2x-3)\cdot 2 = \frac{2\sinh(2x-3)}{1+\cosh^2(2x-3)}$$

Solution (3)

We can rewrite $\left(\tan x\right)^{\ln x+x^2}$ as follows

$$\left(\tan x\right)^{\ln x+x^2} = e^{\ln\left(\tan x\right)\left(\ln x+x^2\right)} \tag{1}$$

then we obtain

$$\left(\left(\tan x\right)^{\ln x+x^2}\right)' = \left(e^{\ln\left(\tan x\right)\left(\ln x+x^2\right)}\right)' = e^{\ln\left(\tan x\right)\left(\ln x+x^2\right)}\left(\ln\left(\tan x\right)\left(\ln x+x^2\right)\right)' =$$

$$= e^{\ln(\tan x)(\ln x+x^2)}\left(\left(\ln(\tan x)\right)'(\ln x+x^2)+\ln(\tan x)(\ln x+x^2)'\right) =$$

$$= e^{\ln(\tan x)(\ln x+x^2)}(\ln'(\tan x)(\tan x)'(\ln x+x^2)+\ln(\tan x)(\ln x+x^2)') =$$

$$= e^{\ln(\tan x)(\ln x+x^2)}\left(\frac{1}{\tan x}\sec^2 x(\ln x+x^2)+\ln(\tan x)\left(\frac{1}{x}+2x\right)\right) =$$

$$= e^{\ln(\tan x)(\ln x+x^2)}\left(\frac{\sec^2 x(\ln x+x^2)}{\tan x}+\ln(\tan x)\left(\frac{1}{x}+2x\right)\right) =$$

And finally, by using (1) we obtain

$$= (\tan x)^{\ln x+x^2}\left(\frac{\sec^2 x(\ln x+x^2)}{\tan x}+\ln(\tan x)\left(\frac{1}{x}+2x\right)\right)$$

Solution (4)

$$\left(\left(\sec^2 x - \tan^2 x\right)^{45}\right)' = 45\left(\sec^2 x - \tan^2 x\right)^{44}\left(\sec^2 x - \tan^2 x\right)' =$$

$$= 45(\sec^2 x - \tan^2 x)^{44} \cdot (2\sec x(\sec x)' - 2\tan x(\tan x)') =$$

$$= 45(\sec^2 x - \tan^2 x)^{44} \cdot (2\sec x \sec x \tan x - 2\tan x \cdot \sec^2 x) =$$

$$= 45(\sec^2 x - \tan^2 x)^{44} \cdot (2\sec^2 x\tan x - 2\sec^2 x\tan x)$$

Answer:

1) $\frac{\ln\left(x^2-3x+8\right)}{\sec\left(x^2+7x\right)} = \frac{(2x-3)}{\left(x^2-3x+8\right)\sec\left(x^2+7x\right)} - \frac{\ln\left(x^2-3x+8\right)\tan\left(x^2+7x\right)(2x+7)}{\sec\left(x^2+7x\right)}$
2) $\arctan\left(\cosh(2x-3)\right) = \frac{2\sinh(2x-3)}{1+\cosh^2(2x-3)}$
3) $\left(\tan x\right)^{\ln x+x^2} = \left(\tan x\right)^{\ln x+x^2}\left(\frac{\sec^2 x\left(\ln x+x^2\right)}{\tan x} + \ln\left(\tan x\right)\left(\frac{1}{x} + 2x\right)\right)$
4) $\left(\sec^2 x - \tan^2 x\right)^{45}$.