

Answer to Question #86385 - Math - Calculus

Question:

After t second, an object is moving at the speed of $t \exp(-t/2)$ meters per second. Express the distance the object travels as a function of time.

Solution:

Let s be the distance in meters traveled by the object at time t seconds.

$$\text{Given speed} = \frac{ds}{dt} = te^{-\frac{t}{2}}$$

$$s = \int \frac{ds}{dt} dt = \int te^{-\frac{t}{2}} dt$$

By applying integration by parts, $\int u dv = uv - \int v du$.

Let $u = t$ and $dv = e^{-\frac{t}{2}} dt$.

$$\text{Then } du = dt \text{ and } v = \frac{e^{-\frac{t}{2}}}{-\frac{1}{2}} = -2e^{-\frac{t}{2}}.$$

$$s = -2te^{-\frac{t}{2}} - \int -2e^{-\frac{t}{2}} dt = -2te^{-\frac{t}{2}} + 2 \left[\frac{e^{-\frac{t}{2}}}{-\frac{1}{2}} \right] + C.$$

Hence distance $s = -2te^{-\frac{t}{2}} - 4e^{-\frac{t}{2}} + C$, where C is an arbitrary constant.