Answer on Question #86381 – Math – Geometry

Question

1. A straight piece of wire of 28cm is cut into two pieces. One piece is bent into a square (i.e. dimensions x times x). The other piece is bent into a rectangle with aspect ratio three (i.e. dimensions y times 3y). What are dimensions, in centimeters, of the square and the rectangle such that the sum of their areas is minimized.

Solution

Put z- length of the first piece of wire, so length of the second piece is equal to $28 - z$.

Side of the square obtain from the equation
$4x = z \Rightarrow x = \frac{1}{4}z,$

area of the square is
$S_1 = x^2 = \left(\frac{1}{4}z\right)^2 = \frac{z^2}{16}.$

Sides of the rectangle were obtained from the equation
$2y + 2 \cdot 3y = 28 - z \Rightarrow y = \frac{1}{8}(28 - z),$

area of the rectangle is
$S_2 = y \cdot 3y = \frac{1}{8}(28 - z) \cdot \frac{3}{8}(28 - z) = \frac{3(28-z)^2}{64}.$

Overall area is
$S = S_1 + S_2 = \frac{z^2}{16} + \frac{3(28-z)^2}{64} = \frac{4z^2+3(784-56z+z^2)}{64} = \frac{4z^2+2352-168z+3z^2}{64} = \frac{7z^2-168z+2352}{64}.$

So we need find z that minimize S. An optimal point can be obtained from the equation $S' = 0$.
$S' = \left(\frac{7z^2-168z+2352}{64}\right)' = \frac{14z-168}{64} = 0 \Rightarrow z = 12.$

$z = 12$ is min cause S' changes its sign from «-» to «+» passing through this point.

In this way
$x = \frac{12}{4} = 3,$
$y = \frac{1}{8}(28 - 12) = 2.$

Answer: dimension of the square is 3x3 cm, dimension of the rectangle is 2x6 cm.