

Answer on Question #86242 – Math – Differential Equations

Question

Solve the following ordinary differential equation:

$$\left\{y\left(1 + \frac{1}{x}\right) + \cos y\right\} dx + (x + \log x - x \sin y) dy = 0$$

Solution

$$M(x, y) = y\left(1 + \frac{1}{x}\right) + \cos y, \frac{\partial M}{\partial y} = 1 + \frac{1}{x} - \sin y$$

$$N(x, y) = x + \log x - x \sin y, \frac{\partial N}{\partial x} = 1 + \frac{1}{x} - \sin y$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

Exact Differential Equation

$$f(x, y) = \int M(x, y) dx = \int \left(y\left(1 + \frac{1}{x}\right) + \cos y\right) dx =$$

$$= y(x + \log x) + x \cos y + g(y)$$

$$\frac{\partial f}{\partial y} = x + \log x - x \sin y + \frac{d}{dy}(g(y)) = N(x, y)$$

$$x + \log x - x \sin y + \frac{d}{dy}(g(y)) = x + \log x - x \sin y$$

$$\frac{d}{dy}(g(y)) = 0$$

$$g(y) = c_1$$

$$f(x, y) = y(x + \log x) + x \cos y + c_1 = c_2$$

Finally, the solution of the ordinary differential equation is given by

$$f(x, y) = C,$$

where $f(x, y) = y(x + \log x) + x \cos y$, C is an arbitrary real constant.