

Answer to Question #86187 - Math – Calculus

Question:

A solid in the shape of a hemisphere with a radius of 2 units, has its base in the xy -plane and the centre of the base at the origin. If the density of the solid is given by the function $\rho(x, y, z) = xyz$, determine the mass of the hemisphere.

Solution:

Mass of the hemisphere $M = \iiint_V \rho dV$

$$M = \int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_0^{\sqrt{4-x^2-y^2}} xyz dz dy dx$$

$$M = \int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} xy \left[\frac{z^2}{2} \right]_0^{\sqrt{4-x^2-y^2}} dy dx$$

$$M = \int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} xy \left[\frac{4-x^2-y^2}{2} \right] dy dx$$

$$M = \int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} x \left[\frac{4y-x^2y-y^3}{2} \right] dy dx \quad \text{Since the integrand is an odd function in } y,$$

$$M = 0.$$

Hence mass of this hemisphere = 0.