

Answer to Question #86177 - Math - Statistics and Probability

Question: The arithmetic mean of 10 numbers is 4, when an eleventh number, x is added so that the overall mean is changed to 5. When a twelfth number, y is added the mean changes to 4. Determine the values of x and y .

Solution: Let z_1, \dots, z_{10} be the initial 10 numbers. Given that, their arithmetic mean is 4.

Therefore,

$$\begin{aligned}\frac{1}{10} \sum_{i=1}^{10} z_i &= 4 \\ \Rightarrow \sum_{i=1}^{10} z_i &= 40\end{aligned}$$

When x is added in this list, the overall mean is 5. So we have,

$$\begin{aligned}\frac{1}{11}(z_1 + \dots + z_{10} + x) &= 5 \\ \Rightarrow (z_1 + \dots + z_{10} + x) &= 55 \\ \Rightarrow 40 + x &= 55 \\ \Rightarrow x &= 15\end{aligned}$$

When y is added in this list as the twelfth number, the overall mean is 4. So we have,

$$\begin{aligned}\frac{1}{12}(z_1 + \dots + z_{10} + x + y) &= 4 \\ \Rightarrow (z_1 + \dots + z_{10} + x + y) &= 48 \\ \Rightarrow 40 + 15 + y &= 48 \\ \Rightarrow y &= -7\end{aligned}$$

Answer: Therefore, the values are,

- $x = 15$
- $y = -7$