Question

There are two coins-one unbiased with P(H)=1/2, the other biased with P(H)=2/5.

One of these coins is selected and tossed 5 times. If the head comes up at least twice, the coin is assumed to be unbiased. Find the level of significance and power of the test.

Solution

Null hypothesis is that coin is unbiased. Alternative hypothesis is that coin is biased.

Level of significance is probability to reject true null hypothesis:

$$\alpha = P_0(less\ than\ two\ heads) =$$
$$= P_0(0\ heads) + P_0(1\ heads) =$$
$$= \left(\frac{1}{2}\right)^5 + \binom{5}{1} \cdot \left(\frac{1}{2}\right) \cdot \left(\frac{1}{2}\right)^4 =$$
$$= 0.03125 + 0.15625 = 0.1875$$

Power of the test is the probability to reject null hypothesis while alternative is true:

$$\alpha = P(less\ than\ two\ heads) =$$
$$= P(0\ heads) + P(1\ heads) =$$
$$= \left(\frac{3}{5}\right)^5 + \binom{5}{1} \cdot \left(\frac{2}{5}\right) \cdot \left(\frac{3}{5}\right)^4 =$$
$$= 0.33696.$$