Answer on Question #86142 – Math – Statistics and Probability

Question

If 20% of the memory chips made in a certain plant are defective, find the probability that in a lot of 100 randomly chosen chips for inspection:
(i) at most 15 chips will be defective
(ii) the number of defectives will be between 15 and 25.

Solution

The binomial distribution $b(x; n, p)$

$$b(x; n, p) = \binom{n}{x} p^x (1-p)^{n-x}$$

$n = 100, p = 0.2$

(i) at most 15 chips will be defective

$$p(x \le 15) = \sum_{i=0}^{15} p(x=i) = \binom{100}{0} p^0 (1-p)^{100-0} + \binom{100}{1} p^1 (1-p)^{100-1} +$$
$$+\binom{100}{2} p^2 (1-p)^{100-2} + \binom{100}{3} p^3 (1-p)^{100-3} + \binom{100}{4} p^4 (1-p)^{100-4} +$$
$$+\binom{100}{5} p^5 (1-p)^{100-5} + \binom{100}{6} p^6 (1-p)^{100-6} + \binom{100}{7} p^7 (1-p)^{100-7} +$$
$$+\binom{100}{8} p^8 (1-p)^{100-8} + \binom{100}{9} p^9 (1-p)^{100-9} + \binom{100}{10} p^{10} (1-p)^{100-10} +$$
$$+\binom{100}{11} p^{11} (1-p)^{100-11} + \binom{100}{12} p^{12} (1-p)^{100-12} +$$
$$+\binom{100}{13} p^{13} (1-p)^{100-13} + \binom{100}{14} p^{14} (1-p)^{100-14} +$$
$$+\binom{100}{15} p^{15} (1-p)^{100-15} = 2.027 \times 10^{-10} + 5.096 \times 10^{-9} + 6.302 \times 10^{-8} +$$
$$+5.147 \times 10^{-7} + 3.120 \times 10^{-6} + 1.498 \times 10^{-5} + 5.928 \times 10^{-5} +$$
$$+1.990 \times 10^{-4} + 5.784 \times 10^{-4} + 0.001478 + 0.003362 + 00.6878 +$$
$$+0.012754 + 0.021583 + 0.033531 + 0.048062 \approx 0.1285$$

(ii) the number of defectives will be between 15 and 25.

$$p(15 \le x \le 25) = \binom{100}{15} p^{15} (1-p)^{100-15} + \binom{100}{16} p^{16} (1-p)^{100-16} +$$
$$+\binom{100}{17} p^{17} (1-p)^{100-7} + \binom{100}{18} p^{18} (1-p)^{100-18} +$$
$$+\binom{100}{19} p^{19} (1-p)^{100-19} + \binom{100}{20} p^{20} (1-p)^{100-20} +$$
$$+\binom{100}{21} p^{21} (1-p)^{100-21} + \binom{100}{22} p^{22} (1-p)^{100-22} +$$
$$+\binom{100}{23} p^{23} (1-p)^{100-23} + \binom{100}{24} p^{24} (1-p)^{100-24} +$$

$$+\binom{100}{25}p^{25}(1-p)^{100-25} = 0.048062 + 0.063832 + 0.078851 +$$
$$+0.090898 + 0.098074 + 0.099300 + 0.094572 + 0.084900 +$$
$$+0.090898 + 0.071980 + 0.057734 + 0.043878 \approx 0.8321$$