Question

If 6 ΦcosΦ + 2 sin²Φ =5, show that tan²Φ - ⅓

Solution

$$6cos\varphi + 2sin^2\varphi = 5;$$

$$6cos\varphi + 2 - 2cos^2\varphi - 5 = 0;$$

$$6cos\varphi - 2cos^2\varphi - 3 = 0;$$

$$2cos^2\varphi - 6cos\varphi + 3 = 0$$

If $x = cos\varphi$ and $-1 \le cos\varphi \le 1$ that $2x^2 - 6x + 3 = 0$;

$$2x^2 - 6x + 3 = 0;$$

$$x = \frac{6 \pm \sqrt{36 - 4*2*3}}{2*2};$$

$$x = 1.5 - 0.5\sqrt{3};$$

$x = 1.5 + 0.5\sqrt{3} -$ this solution is not included in the range $-1 \le cos\varphi \le 1$

Then $cos\varphi = 1.5 - 0.5\sqrt{3}$;

$tan^2\varphi + 1 = \frac{1}{cos^2\varphi}$;

$tan^2\varphi = \frac{1}{cos^2\varphi} - 1 = \frac{1}{\left(1.5 - 0.5\sqrt{3}\right)^2} - 1$ and it is not equal to $\frac{1}{3}$.

P.S. The task probably contains a typo ΦcosΦ since that equation can be solved with a help of the graphical method and numerical approximations.