Answer to the question 86083, Math / Algebra.

Use the principle of induction to prove the statement: $2^n > 1 + n\sqrt{2^{n-1}}$ for all $n > 2$.

For $n = 3$ we have $2^n = 8$, $n\sqrt{2^{n-1}} = 3\sqrt{4} = 6$, $8 > 6$. Thus the inequality holds for $n = 3$ and we proved the base case.

Suppose that $2^n > 1 + n\sqrt{2^{n-1}}$. Let us prove that

$$2^{n+1} > 1 + (n+1)\sqrt{2^n}.$$

We have $2^{n+1}/2^n = 2$,

$$\frac{1 + (n+1)\sqrt{2^n}}{1 + n\sqrt{2^{n-1}}} < \frac{n+1}{n}\sqrt{2}.$$

Thus it is enough to prove that for $n \geq 3$

$$\frac{n+1}{n} < \sqrt{2},$$

$$\left(\frac{n+1}{n}\right)^2 < 2,$$

$$\frac{n^2 + 2n + 1}{n^2} < 2,$$

$$2n + 1 < n^2,$$

$$n^2 - 2n + 1 - 2 > 0,$$

$$(n-1)^2 > 2.$$

It is clear that for $n \geq 3$

$$(n-1)^2 \geq (3-1)^2 = 4 > 2.$$

Thus we proved the induction step.

The statement is proved using the principle of induction.