

Answer to Question #86080 – Math – Algebra

Question

Give a direct proof, as well as a proof by contradiction, of the following statement:
A intersection B is contained in A union B for any two sets A and B.

Solution

Direct Proof:

Let $x \in A \cap B$

$$\Rightarrow x \in A \Rightarrow x \in A \text{ or } x \in B \Rightarrow x \in A \cup B .$$

Thus,

$$A \cap B \subset A \cup B .$$

Proof by Contradiction:

Suppose $A \cap B \not\subset A \cup B$, so there must be some $x \in A \cap B$ such that $x \notin A \cup B$.

From $x \in A \cap B$ it follows that $x \in A$ and $x \in B$.

But, from $x \notin A \cup B$ it follows that $x \notin A$ and $x \notin B$, which contradicts the former implication. Thus, an assumption $A \cap B \not\subset A \cup B$ was false and

$$A \cap B \subset A \cup B .$$