Answer on Question #86079 – Math – Algebra

Question

Find the polynomial equation over R of lowest degree which is satisfied by (1-i) and (3+2i).

Solution

When the polynomial equation over R has an integrated root, then the conjugate number to the root is also the root. So $(1+i)$ and $(3-2i)$ are the roots of the equation.

Since we have 4 roots, the polynomial equation over R of lowest degree is of the fourth degree. Then

$$\left(x-(1-i)\right)\left(x-(1+i)\right)\left(x-(3+2i)\right)\left(x-(3-2i)\right)=0,$$

$$\left(x^2-(1+i)x-(1-i)x+(1-i)(1+i)\right)\left(x^2-(3-2i)x-(3+2i)x+(3+2i)(3-2i)\right)=$$
$$=0,$$

$$(x^2-2x+1-i^2)(x^2-6x+9-4i^2)=0,$$

$$(x^2-2x+2)(x^2-6x+13)=0,$$

$$x^4-6x^3+13x^2-2x^3+12x^2-26x+2x^2-12x+26=0,$$

$$x^4-8x^3+27x^2-38x+26=0.$$

Answer: $x^4-8x^3+27x^2-38x+26=0$ is a polynomial equation over R of the lowest degree which is satisfied by (1-i) and (3+2i).