Answer on Question #85964 – Math – Discrete Mathematics

Question

Recall that a real number x is rational if x = p/q for integers p, q with $q \neq 0$.

Prove that if x is rational then $1/(2x+1)$ is rational. Then prove that if 1/(2x+1) is rational then x is rational.

Solution

1) If ***x*** is rational then ***x*** = $\frac{p}{q}$**,** where ***p*** and ***q*** are integers with ***q*** ≠ 0.

So $\frac{1}{2x+1} = \frac{1}{2\frac{p}{q}+1} = \frac{1}{\frac{2p+q}{q}} = \frac{q}{2p+q}$. Since ***q*** and ***2p + q*** are integers with ***2p + q*** ≠ 0, then $\frac{q}{2p+q} = \frac{1}{2x+1}$ is rational.

2) If $\frac{1}{2x+1}$ is rational then $\frac{1}{2x+1}$ = $\frac{p}{q}$**,** where ***p*** and ***q*** are integers with ***q*** ≠ 0.

In addition, ***p*** = ***q*** $* \frac{1}{2x+1} \neq 0$. Therefore ***2x + 1*** = $\frac{q}{p}$, and ***2x*** = $\frac{q}{p} - 1 = \frac{q-p}{p}$. So ***x*** = $\frac{q-p}{2p}$**.**

Since ***q - p*** and ***2p*** are integers with ***2p*** ≠ 0, then $\frac{q-p}{2p} = x$ is rational.

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