Answer on Question #85937 – Math – Linear Algebra

Question

Which of the following are subspaces of R^3? Justify your answer.
1) S={(x,y,z)€R^3 | x+y=z }
2) S={(x,y,z)€R^3 | 2x=3yz }

Solution

The subset of linear space is a subspace if it is a linear space. A subspace is a closed set with respect to addition and multiplication by scalar. Let's check it:

1) If $(x_1, y_1, z_1), (x_2, y_2, z_2) \in S$ that $x_1+y_1 = z_1$ and $x_2+y_2 = z_2$, then $(x_1, y_1, z_1)+(x_2, y_2, z_2) = (x_1+x_2, y_1+y_2, z_1+z_2)$; $(x_1+x_2)+(y_1+y_2) = (x_1+y_1)+(x_2+y_2) = z_1+z_2$. So $(x_1, y_1, z_1)+(x_2, y_2, z_2) \in S$.

Let $(x, y, z) \in S$ that $x+y = z$; $\alpha(x, y, z) = (\alpha x, \alpha y, \alpha z)$, $\alpha x+\alpha y = \alpha(x+y) = \alpha z$, that is, $(\alpha x, \alpha y, \alpha z) \in S$. Thus, S is a subspace of $\mathbb{R}^3$.

2) Let $(x, y, z) \in S$ that $2x = 3yz$. $\alpha(x, y, z) = (\alpha x, \alpha y, \alpha z)$, $2\alpha x = \alpha(2x) = \alpha(3yz) = 3(\alpha yz) \neq 3(\alpha y \alpha z)$ that is $\alpha(x, y, z) \notin S$. Thus, S is not a subspace of $\mathbb{R}^3$.