Answer on Question #85921 – Math – Statistics and Probability

Question

A manufacturing firm produces steel pipes in three plants with daily production volume of 500, 1000 and 2000 units, respectively. According to past experience, it is known that the fraction of defective outputs produced by the three plants are 0.005, 0.008 and 0.010, respectively. If a pipe is selected from a day's total production and found to be defective, then
(i) find the probability that it came from the first plant,
(ii) identity most likely plant which has produced it.

Solution

Assume that a manufacturing firm produces steel pipes in three plants A, B and C with daily production volume of 500, 1000 and 2000 units, respectively.
We have probability of production

$$P(A) = \frac{500}{500 + 1000 + 2000} = \frac{1}{7}$$

$$P(B) = \frac{1000}{500 + 1000 + 2000} = \frac{2}{7}$$

$$P(C) = \frac{2000}{500 + 1000 + 2000} = \frac{4}{7}$$

Probability of defective pipes

$$P(A|E) = 0.005$$

$$P(B|E) = 0.008$$

$$P(C|E) = 0.010$$

(i) By Bayes' Theorem, probability of defective pipe from the first plant

$$P(E|A) = \frac{P(A)P(A|E)}{P(A)P(A|E) + P(B)P(B|E) + P(C)P(C|E)}$$

$$P(E|A) = \frac{\frac{1}{7}(0.005)}{\frac{1}{7}(0.005) + \frac{2}{7}(0.008) + \frac{4}{7}(0.010)} = \frac{5}{61}$$

(ii) Probability of defective pipe from the second plant

$$P(E|B) = \frac{P(B)P(B|E)}{P(A)P(A|E) + P(B)P(B|E) + P(C)P(C|E)}$$

$$P(E|B) = \frac{\frac{2}{7}(0.008)}{\frac{1}{7}(0.005) + \frac{2}{7}(0.008) + \frac{4}{7}(0.010)} = \frac{16}{61}$$

Probability of defective pipe from the third plant

$$P(E|C) = \frac{P(C)P(C|E)}{P(A)P(A|E) + P(B)P(B|E) + P(C)P(C|E)}$$

$$P(E|C) = \frac{\frac{4}{7}(0.010)}{\frac{1}{7}(0.005) + \frac{2}{7}(0.008) + \frac{4}{7}(0.010)} = \frac{40}{61}$$

The defective pipe comes most likely from the third plant.