**Answer on Question #85906 – Math – Discrete Mathematics**

## Question

Prove by mathematical induction the formula

$$1^3 + 2^3 + 3^3 + 4^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$$

## Solution

For any integer $n \geq 1$, let $P_n$ be the statement that

$$1^3 + 2^3 + 3^3 + 4^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$$

Base case. The statement $P_1$ says that

$$1^3 = \frac{1^2(1+1)^2}{4},$$

which is true.

Inductive case. Fix $k \geq 1$, and suppose that $P_k$ holds, that is,

$$1^3 + 2^3 + 3^3 + 4^3 + \cdots + k^3 = \frac{k^2(k+1)^2}{4}$$

It remains to show that $P_{k+1}$ holds, that is,

$$1^3 + 2^3 + 3^3 + 4^3 + \cdots + (k+1)^3 = \frac{(k+1)^2\big((k+1)+1\big)^2}{4}$$

$1^3 + 2^3 + 3^3 + 4^3 + \cdots + (k+1)^3 =$

$= 1^3 + 2^3 + 3^3 + 4^3 + \cdots + k^3 + (k+1)^3 =$

$= \dfrac{k^2(k+1)^2}{4} + (k+1)^3 =$

$= (k+1)^2\left(\dfrac{k^2}{4} + k + 1\right) =$

$= \dfrac{(k+1)^2}{4}(k^2 + 4k + 4) =$

$= \dfrac{(k+1)^2}{4}(k+2)^2 =$

$= \dfrac{(k+1)^2\big((k+1)+1\big)^2}{4}$

Therefore, $P_{k+1}$ holds.

Therefore, by the principle of mathematical induction, for all $n \geq 1, P_n$ holds

$$1^3 + 2^3 + 3^3 + 4^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}, n \in \mathbb{N}.$$