Question

An explosion in a factory manufacturing explosive can occur because of (i) leakage of electricity, (ii) defects in machinery, (iii) carelessness of workers and (iv) conspiracy, whose probabilities are respectively 0.25, 0.2, 0.5 and 0.75. The probability that
(i) there is a leakage of electricity is 0.2.
(ii) the machinery is defective is 0.3.
(iii) the workers are careless is 0.4.
(iv) there is a conspiracy is 0.1.
Which is the most likely cause of explosion?

Solution

Suppose the sample space Ω consists of manufacturing accidents caused by the reasons i-iv.
Let $A_1, \dots, A_4$ denote the accidents caused by every reason, so they constitute a partition of Ω. Let B denotes the explosure event.
We are given that $P(A_1) = 0.2$, $P(A_2) = 0.3$, $P(A_3) = 0.4$, $P(A_4) = 0.1$, $P(B|A_1) = 0.25, P(B|A_2) = 0.2$, $P(B|A_3) = 0.5, P(B|A_4) = 0.75$.
Obviously, by Bayes' rule,

$$P(A_i|B) = \frac{P(A_i)P(B|A_i)}{\sum P(A_j)P(B|A_j)}$$

Because we need to determine which value $P(A_i|B)$ is the largest one, we notice that the **non-zero** denominator $\sum P(A_j)P(B|A_j)$ will be the same while calculating each $P(A_i|B)$.
So, consider $P(A_1)P(B|A_1) = 0.05, P(A_2)P(B|A_2) = 0.06, P(A_3)P(B|A_3) = 0.2$, $P(A_4)P(B|A_4) = 0.075$. Among these expressions the value of $P(A_3)P(B|A_3)$ is largest, hence $P(A_3|B)$ will be greater than $P(A_1|B)$, $P(A_2|B)$, $P(A_4|B)$.
We may conclude that the reason iii) is the most likely cause of explosion.

Answer:

Carelessness of workers is the most likely cause of explosion.