

Answer on Question #85800 – Math – Discrete Mathematics

Question

Prove that for all integers n , $n(n + 2)(n + 4)$ is divisible by 3.

Solution

There are only 3 possible remainders when dividing by 3, namely 0, 1, 2.

If n has the remainder 0 (that is, $n \bmod 3 = 0$), it means that n is divisible by 3 and hence $n(n + 2)(n + 4)$ is divisible by 3 too because n is a multiplier of the expression $n(n + 2)(n + 4)$.

If n has the remainder 1 (that is, $n \bmod 3 = 1$), then $(n+2)$ is divisible by 3 because $(n+2)$ has the remainder $1+2 = 3$ and it is the same as to have the remainder $0 \bmod 3$, therefore $n(n + 2)(n + 4)$ is divisible by 3 too.

If n has the remainder 2 (that is, $n \bmod 3 = 2$), then $(n+4)$ is divisible by 3 because $(n+4)$ has the remainder $2+4 = 6$ and it is the same as to have the remainder $0 \bmod 3$, therefore $n(n + 2)(n + 4)$ is divisible by 3 too.

Thus, in any case $n(n + 2)(n + 4)$ is divisible by 3.