

Answer on Question #85744 – Math – Algebra

Question

Check whether the set of vectors $v_1 = (1, 1, 0, 1)$, $v_2 = (1, 0, 2, 1)$, $v_3 = (-1, 1, -3, -2) \in \mathbb{R}^4$ are linearly independent. If they are dependent, find a_1, a_2 and $a_3 \in \mathbb{R}$, not all zero, such that $a_1v_1 + a_2v_2 + a_3v_3 = 0$.

Solution

Determine if the set of vectors is linearly independent solving the following equation:

$$a_1 \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} + a_2 \begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix} + a_3 \begin{bmatrix} -1 \\ 1 \\ -3 \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

or what is the same, the set of equations:

$$\begin{cases} a_1 + a_2 - a_3 = 0 & (1) \\ a_1 + a_3 = 0 & (2) \\ 2a_2 - 3a_3 = 0 & (3) \\ a_1 + a_2 - 2a_3 = 0 & (4) \end{cases}$$

Subtract (4) from (1):

$$\begin{cases} a_1 + a_2 - a_3 = 0 \\ a_1 + a_3 = 0 \\ 2a_2 - 3a_3 = 0 \\ a_3 = 0 \end{cases}$$

$$\begin{cases} a_1 + a_2 = 0 \\ a_1 = 0 \\ 2a_2 = 0 \\ a_3 = 0 \end{cases}$$

$$\begin{cases} a_2 = 0 \\ a_1 = 0 \\ a_2 = 0 \\ a_3 = 0 \end{cases}$$

As $(a_1, a_2, a_3) = (0, 0, 0)$ is the only solution of the system then the given vectors are linearly independent.

Answer: the vectors are linearly independent.