Answer on Question #85733 – Math – Calculus

Question

Apply implicit function theorem to the equation $x^3 + y^3 - 6xy + 4 = 0$ and show that the equation defines a unique function g defined in a neighbourhood of the point $(1,1)$ such that $g(1) = 1$. Also find g'.

Solution

The function $F(x,y) = x^3 + y^3 - 6xy + 4$ is a polynomial function, and therefore a continuously differentiable function. The point $p = (1,1)$ satisfies the equation: $1 + 1 - 6 + 4 = 0$. The derivative $\partial_y F = 3y^2 - 6x$ is not zero at p: $3 - 6 \neq 0$. Thus, $F(x,y)$ meets all conditions of implicit function theorem, and implicit function $y = g(x)$ exists in a neighbourhood of p.
Also by the theorem, $g'(x) = -\,\partial_x F / \partial_y F\,\big(x, g(x)\big) = -(3x^2 - 6g)/(3g^2 - 6x)$.

Answer:

$g' = -(3x^2 - 6g)/(3g^2 - 6x)$.