Answer on Question #85729 – Math – Calculus

Question

Use Lagranges Multipliers method to prove that the rectangle of perimeter 4 with largest area is a unit square.

Solution



a) S = xy.

$$\begin{split} P &= 2x + 2y = 4 \Longrightarrow x + y = 2 \Longrightarrow x + y - 2 = 0 \Longrightarrow L(x, y, \lambda) = xy + \lambda(x + y - 2). \\ L'_x &= y + \lambda = 0, L'_y = x + \lambda = 0, L'_\lambda = x + y - 2 = 0 \Longrightarrow \begin{cases} y + \lambda = 0 \\ x + \lambda = 0 \\ x + y - 2 = 0 \end{cases} \Longrightarrow \begin{cases} x = 1 \\ y = 1 \\ \lambda = -1 \end{cases} \\ L''_{xx} &= 0, L''_{xy} = 1, L''_{x\lambda} = 1; L''_{yy} = 0, L''_{y\lambda} = 1, L''_{\lambda\lambda} = 0 \\ maxS(x, y) = L(1, 1, -1) = 1 \cdot 1 - 1 \cdot (1 + 1 - 2) = 1. \end{cases} \\ b)S &= xy \le \left(\frac{x + y}{2}\right)^2 = \left(\frac{2}{2}\right)^2 = 1 \Longrightarrow maxS(x, y) = 1. \end{split}$$

Answer:

$$maxS(x, y) = max(x \cdot y) \begin{vmatrix} x, y > 0 \\ x + y = 2 \end{vmatrix} = 1.$$

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