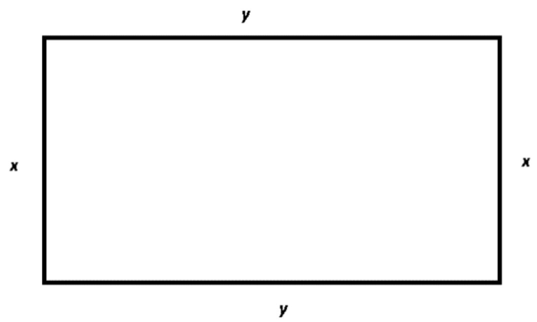
Answer on Question #85729 – Math – Calculus

Question

Use Lagranges Multipliers method to prove that the rectangle of perimeter 4 with largest area is a unit square.

Solution



$$a)\ S = xy.$$

$$P = 2x + 2y = 4 \Longrightarrow x + y = 2 \Longrightarrow x + y - 2 = 0 \Longrightarrow L(x, y, \lambda) = xy + \lambda(x + y - 2).$$

$$L'_x = y + \lambda = 0, L'_y = x + \lambda = 0, L'_\lambda = x + y - 2 = 0 \Longrightarrow \begin{cases} y + \lambda = 0 \\ x + \lambda = 0 \\ x + y - 2 = 0 \end{cases} \Longrightarrow \begin{cases} x = 1 \\ y = 1 \\ \lambda = -1 \end{cases}$$

$$L''_{xx} = 0, L''_{xy} = 1, L''_{x\lambda} = 1; L''_{yy} = 0, L''_{y\lambda} = 1, L''_{\lambda\lambda} = 0$$

$$maxS(x, y) = L(1,1,-1) = 1 \cdot 1 - 1 \cdot (1 + 1 - 2) = 1.$$

$$b) S = xy \le \left(\frac{x + y}{2}\right)^2 = \left(\frac{2}{2}\right)^2 = 1 \Longrightarrow maxS(x, y) = 1.$$

Answer:

$$maxS(x, y) = max(x \cdot y)\Bigg|_{\substack{x, y > 0 \\ x + y = 2}} = 1.$$