Answer on Question #85728 – Math – Calculus

Question

Find the Taylor polynomials of f(x,y) = 2 + 3x + 5y at (0,1).

Solution

n^{th}-degree Taylor polynomial for a function of 2 variables near the point (x_0,y_0) is:

$$T_n(x,y) = \sum_{k=0}^{n} \frac{1}{k!}\left((x-x_0)\frac{\partial}{\partial x} + (y-y_0)\frac{\partial}{\partial y}\right)^n f(x_0,y_0)$$

$$= f(x_0,y_0) + (x-x_0)\frac{\partial f(x_0,y_0)}{\partial x} + (y-y_0)\frac{\partial f(x_0,y_0)}{\partial y}$$

$$+\frac{1}{2}\left((x-x_0)^2\frac{\partial^2 f(x_0,y_0)}{\partial x^2} + (x-x_0)(y-y_0)\frac{\partial^2 f(x_0,y_0)}{\partial x\partial y}\right.$$

$$\left.+ (x-x_0)(y-y_0)\frac{\partial^2 f(x_0,y_0)}{\partial y\partial x} + (y-y_0)^2\frac{\partial^2 f(x_0,y_0)}{\partial y^2}\right) + \cdots$$

The Taylor polynomial of *f(x,y)=2+3x+5y* near point (0,1) (***n*** more or equal 1):

$$f(x,y) = 2 + 3x + 5y \ \rightarrow \ second\ order\ derivatives\ (and\ higher)\ are\ zero$$

$$f(0,1) = 7, \qquad \frac{\partial f(0,1)}{\partial x} = 3, \qquad \frac{\partial f(0,1)}{\partial y} = 5$$

$$T_n(x,y) = f(0,1) + (x-0)\frac{\partial f(0,1)}{\partial x} + (y-1)\frac{\partial f(0,1)}{\partial y} = 7 + 3x + 5y - 5 = 2 + 3x + 5y$$