

Answer on Question #85727 – Math – Calculus

Question

Let $f(x) = (\pi x^3 - 5x + 7)/(2x^3 + 7x - 9)$; and $g(x) = \sin x$ be two real valued functions of a real variable defined on $]2, \infty[$ such that h is their composite function $g \circ f$. Evaluate $\lim_{x \rightarrow \infty} h(x)$

Solution

If $f(x) = \frac{\pi x^3 - 5x + 7}{2x^3 + 7x - 9}$ and $g(x) = \sin x$, then $h(x) = g \circ f = g(f(x)) = \sin\left(\frac{\pi x^3 - 5x + 7}{2x^3 + 7x - 9}\right)$.

$$\begin{aligned} \text{Since the function } g(x) = \sin x \text{ is continuous then } \lim_{x \rightarrow \infty} (h(x)) &= \lim_{x \rightarrow \infty} \left(\sin\left(\frac{\pi x^3 - 5x + 7}{2x^3 + 7x - 9}\right) \right) = \\ &= \sin\left(\lim_{x \rightarrow \infty} \left(\frac{\pi x^3 - 5x + 7}{2x^3 + 7x - 9}\right)\right) = \sin\left(\lim_{x \rightarrow \infty} \left(\frac{\frac{\pi x^3}{x^3} - \frac{5x}{x^3} + \frac{7}{x^3}}{\frac{2x^3}{x^3} + \frac{7x}{x^3} - \frac{9}{x^3}}\right)\right) = \sin\left(\lim_{x \rightarrow \infty} \left(\frac{\pi - \frac{5}{x^2} + \frac{7}{x^3}}{2 + \frac{7}{x^2} - \frac{9}{x^3}}\right)\right) = \\ &= \sin\left(\frac{\pi - 0 + 0}{2 + 0 - 0}\right) = \sin\left(\frac{\pi}{2}\right) = 1 \end{aligned}$$

Answer:

$$\lim_{x \rightarrow \infty} (h(x)) = 1$$