Answer on Question #85701 – Math – Calculus

Question

Trace the curve

$y^2 = (x+1)\ (x-1)^2$ (1)

by showing all the properties you use to trace it.

Solution

1. Domain:

$x + 1 > 0 \implies x > -1 \implies x \in (-1; \infty)$.

2. Symmetrical:

The curve is symmetric about y-axis because at y>0 $and\ y < 0$ the left and right sides of this curve do not change the sign.

3. Not periodic.
4. Points of intersection with axes of coordinates:

$y^2 = (x+1)\ (x-1)^2 \Leftrightarrow y = \pm\sqrt{x+1}|x-1|$ (2)

Ox: $f(x) = 0 \implies x = -1$ and $x = 1 \implies$ (-1; 0) and (1; 0) – wanted points;

Oy: $x = 0 \implies f(0) = \pm 1 \implies$(0; -1) and (0; 1).

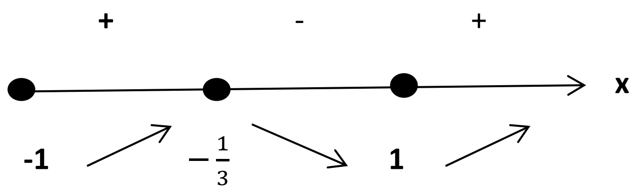
5. Extremums and monotony intervals:

$$(y^2)^{/} = \left((x+1)\ (x-1)^2\right)^{/} \implies 2y\, y^{/} = (x-1)^2 + 2(x-1)(x+1) \implies$$

$$y^{/} = \frac{(x-1)(3x+1)}{2y} = \frac{(x-1)(3x+1)}{\pm 2\sqrt{x+1}|x-1|}$$

$y^{/} = 0 \implies \frac{(x-1)(3x+1)}{\pm 2\sqrt{x+1}|x-1|} = 0 \implies 3x + 1 = 0 \implies x = -\frac{1}{3}, x \neq -1, x \neq 1$

a) $y^{/} = \frac{(x-1)(3x+1)}{2\sqrt{x+1}|x-1|}$ for $y = \sqrt{x+1}|x-1|$



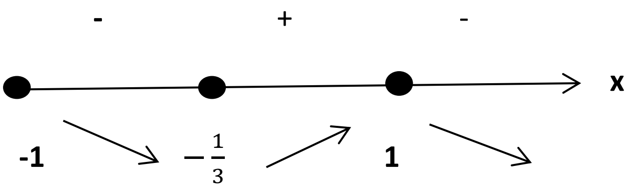
x $\in$ (-1; $-\frac{1}{3}$) - the function is monotonously increasing;

x ϵ (-$\frac{1}{3}$; 1) - the function is monotonously decreases;

x ϵ (1;+∞) - the function is monotonously increasing;

(-$\frac{1}{3}$; $\frac{4\sqrt{6}}{9}$) is the point of maximum and (1;0) is the point of minimum.

b) $y' = -\frac{(x-1)(3x+1)}{2\sqrt{x+1}|x-1|}$ for $y = -\sqrt{x+1}|x-1|$



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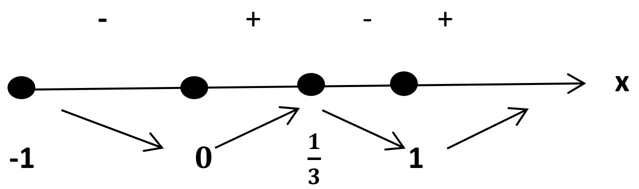
(-$\frac{1}{3}$; $-\frac{4\sqrt{6}}{9}$) is the point of minimum and (1; 0) is the point of maximum .

6. Points of overhang and concavity:

$$f(x)'' = 0.$$

a) $y' = \frac{(x-1)(3x+1)}{2\sqrt{x+1}|x-1|}$, $y'' = \frac{(x-1)(3x-1)x}{4(x+1)\sqrt{x+1}|x-1|}$ for $y = \sqrt{x+1}|x-1| \Longrightarrow \frac{(x-1)(3x-1)x}{4(x+1)\sqrt{x+1}|x-1|} = 0$

$$\Longrightarrow x = 0, x = \frac{1}{3}, x \neq 1, x \neq -1.$$



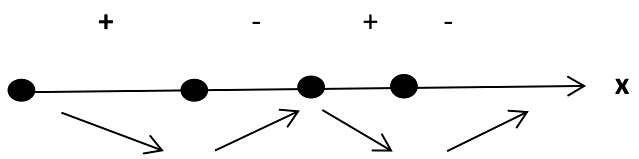
x ϵ (-1;0) and ($\frac{1}{3}$;1)- the curve is convex,

x ϵ (0;$\frac{1}{3}$) and (1;+∞)- the curve is curved.

b) $y' = -\frac{(x-1)(3x+1)}{2\sqrt{x+1}|x-1|}$,

$y'' = -\frac{(x-1)(3x-1)x}{4(x+1)\sqrt{x+1}|x-1|}$ for $y = -\sqrt{x+1}|x-1|$ $\Longrightarrow$ $-\frac{(x-1)(3x-1)x}{4(x+1)\sqrt{x+1}|x-1|} = 0$

$$\Longrightarrow x = 0, x = \frac{1}{3}, x \neq 1, x \neq -1.$$



-1 0 $\frac{1}{3}$ 1

x ϵ (-1;0) and ($\frac{1}{3}$;1)- the curve is curved,

x ϵ (0;$\frac{1}{3}$) and (1;+∞)- the curve is convex.

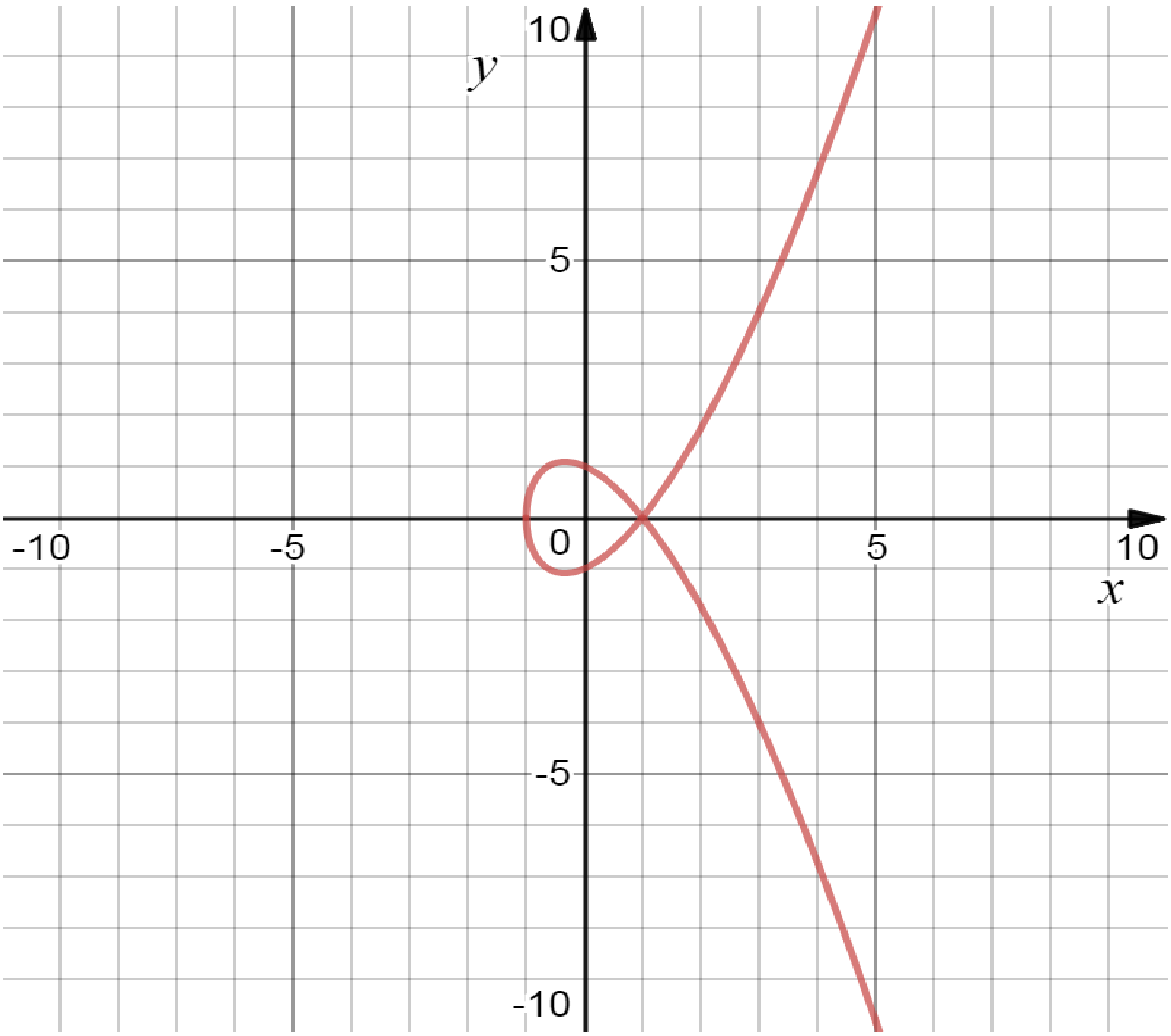
7. Asymptotes:

a) horizontal asymptotes is not so $\lim_{x\to\infty} f(x) = \lim_{x\to\infty}(\pm\sqrt{x+1}|x-1|) = \infty$, $\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty}(\pm\sqrt{x+1}|x-1|) -$ does not exist;

b) vertical asymptotes is not so (1) is continuous at x ϵ (-1;+∞) ;

c) inclined asymptotes is not so $\lim_{x\to\pm\infty}\frac{\pm\sqrt{x+1}|x-1|}{x} = \pm\infty$.

8. We build the function (1) graph (use research 1-7):



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