Answer to Question #85696 – Math – Calculus

Y=Sinh⁻¹(tanhx)

If Y=1/Sinh(tanhx), then

$$Y = \left[\frac{2}{e^{\tanh x} - e^{-\tanh x}} \right]$$

Differentiate wrt x

$$\frac{dY}{dx} = \frac{\left(e^{\tanh x} - e^{-\tanh x}\right)\dfrac{d}{dx}(2) - 2\dfrac{d}{dx}\left(e^{\tanh x} - e^{-\tanh x}\right)}{\left(e^{\tanh x} - e^{-\tanh x}\right)^2}$$

$$\frac{dY}{dx} = \frac{\left(e^{\tanh x} - e^{-\tanh x}\right)0 - 2\left(e^{\tanh x} \times \sec h^2 x + e^{-\tanh x} \times \sec h^2 x\right)}{\left(e^{\tanh x} - e^{-\tanh x}\right)^2}$$

$$\frac{dY}{dx} = \frac{2\left(e^{\tanh x} + e^{-\tanh x}\right)}{\left(e^{\tanh x} - e^{-\tanh x}\right)^2} \sec h^2 x$$

$$\frac{dY}{dx} = \frac{2\left(e^{\tanh x} + e^{-\tanh x}\right)}{\left(e^{\tanh x} - e^{-\tanh x}\right)^2} \left(\frac{2}{e^x + e^{-x}}\right)^2$$

If the inverse function is meant, then

Let Y=Sinh⁻¹(tanhx)

Sinhy=tanhx

Differentiate both side wrt to x

coshy dy/dx=sec²hx

dy/dx=sechy*sec²hx.