Answer on Question #85606 – Math – Statistics and Probability

Question

A continuous random variable X has moment generating function $M(t) = e^{2t^2+3t}$.
Determine the $E(X)$ and at most two errors.

Solution

$E(X) = \dfrac{d}{dt}M(t)|_{t=0}$

$M(t) = e^{2t^2+3t}$

$\dfrac{d}{dt}M(t) = \dfrac{d}{dt}\left(e^{2t^2+3t}\right) = e^{2t^2+3t}(4t+3)$

$E(X) = \dfrac{d}{dt}M(t)|_{t=0} = e^{0+0}(4(0)+3) = 3$

We have normal distribution

$M(t) = e^{\frac{\sigma^2}{2}t^2+\mu t}$

$\mu = 3, \sigma^2 = 4$

95% of the data is within 2 standard deviations (σ) of the mean (μ)

$P(\mu - 2\sigma \le X \le \mu + 2\sigma) = 0.9545$

$Z = \dfrac{X - \mu}{\sigma}$

$Z = \dfrac{2 - 3}{\sqrt{4}} = -0.5$

$P(X \le 2) = P(Z \le -0.5) = 0.3085$

Answer: 3; 0.3085.