

Answer on Question #85605 – Math – Statistics and Probability

Question

In 500 independent calculations, a student made 25 errors. His instructor randomly checked seven calculations of the student. Find the probability that instructor detects

- i) Exactly 2 errors
- ii) At most two errors

Solution

Let p is the probability that instructor detects error in randomly selected calculation of the student. Then $p = \frac{25}{500} = 0.05$.

The probability that instructor detects in n calculations exactly k errors is equal to

$$P_n(k) = \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k},$$

- i) In our problem $n = 7$, $k = 2$, $p = 0.05$.

$$\text{So } P_7(2) = \frac{7!}{2!(7-2)!} * 0.05^2 * (1 - 0.05)^{7-2} = 21 * 0.05^2 * 0.95^5 = 0.04062$$

- ii) The probability that instructor detects in 7 calculations at most two errors is equal to

$$\begin{aligned} P_7(\leq 2) &= P_7(2) + P_7(1) + P_7(0) = \frac{7!}{2!(7-2)!} * 0.05^2 * (1 - 0.05)^{7-2} + \\ &+ \frac{7!}{1!(7-1)!} * 0.05^1 * (1 - 0.05)^{7-1} + \frac{7!}{0!(7-0)!} * 0.05^0 * (1 - 0.05)^{7-0} = \\ &= 0.04062 + 7 * 0.05^1 * 0.95^6 + 1 * 1 * 0.95^7 = 0.04062 + 0.25728 + 0.69834 = \\ &= 0.9962 \end{aligned}$$

Answer:

- i) The probability that instructor detects in 7 calculations exactly 2 errors is equal to 0.04062.
- ii) The probability that instructor detects in 7 calculations at most two errors is equal to 0.9962.