

Answer on Question #85528 – Math – Calculus

Question

If $f(x) = \frac{4x^2 - 7x - 2}{x - 2}$, x not equal to 2, find a $z > 0$ such that $|f(x) - 9| < (1/100)$ for $0 < |x - 2| < z$. Hence, show that $\lim_{x \rightarrow 2} f(x) = 9$.

Solution

Let $f(x) = \frac{4x^2 - 7x - 2}{x - 2}$. If x is not equal to 2, then $f(x) = \frac{4x^2 - 7x - 2}{x - 2} = \frac{(x - 2)(4x + 1)}{x - 2} = 4x + 1$.

Therefore, $|f(x) - 9| = |4x + 1 - 9| = |4x - 8| = 4 * |x - 2|$.

If $|f(x) - 9| = 4 * |x - 2| < 0.01$, then $|x - 2| < \frac{0.01}{4}$. Hence, $z = 0.0025$.

Therefore, if $z = \frac{d}{4}$, then from the fact that $|x - 2| < z$, it follows that $|f(x) - 9| < d$.

It means that $\lim_{x \rightarrow 2} \frac{4x^2 - 7x - 2}{x - 2} = 9$.

Answer:

$$z = 0.0025$$