

Answer on Question #85526 – Math – Statistics and Probability

Question

Let X be a continuous random variable with probability density function given by

$$f(x) = \begin{cases} \frac{1}{2}x, & \text{for } 0 \leq x \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

Determine the mean and variance of this distribution.

Solution

The mean is

$$\mu = E(X) = \int_{-\infty}^{\infty} xf(x) dx = \int_0^2 \frac{1}{2}x^2 dx = \left[\frac{x^3}{6} \right]_0^2 = \frac{8}{6} - 0 = \frac{4}{3}$$

The variance is

$$\begin{aligned} V(X) &= \sigma^2 = E((X - \mu)^2) = \int_{-\infty}^{\infty} \left(x - \frac{4}{3}\right)^2 f(x) dx = \int_0^2 \frac{1}{2}x \left(x - \frac{4}{3}\right)^2 dx = \\ &= \frac{1}{2} \int_0^2 \left(x^3 - \frac{8}{3}x^2 + \frac{16}{9}x\right) dx = \frac{1}{2} \left[\frac{x^4}{4} - \frac{8}{9}x^3 + \frac{8}{9}x^2 \right]_0^2 = \\ &= \frac{1}{2} \left(4 - \frac{64}{9} + \frac{32}{9} \right) = \frac{2}{9} \end{aligned}$$

Or

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x) dx = \int_0^2 \frac{1}{2}x^3 dx = \left[\frac{x^4}{8} \right]_0^2 = \frac{16}{8} - 0 = 2$$

$$V(X) = E(X^2) - (E(X))^2 = 2 - \left(\frac{4}{3}\right)^2 = \frac{2}{9}$$

Answer: $\frac{4}{3}, \frac{2}{9}$.