Answer to Question #85521 - Math - Statistics and Probability

Question: Let X be a continuous random variable with probability density function given by,

$$f(x) = \begin{cases} \frac{1}{2}x, \text{ for } 0 \leq x \leq 2 \\ 0, \text{ otherwise} \end{cases}$$

Determine the mean and variance of this distribution.

Solution: First we need to check that the function f given in the question is a density function.

- $\forall x \in \mathbb{R}, f(x) \geq 0$
- $$\int_{-\infty}^{\infty} f(x)dx = \int_0^2 \frac{1}{2}xdx = 1$$

Therefore, f is a density.

Calculating the mean:

$$\begin{aligned} E(X) &= \int_{-\infty}^{\infty} xf(x)dx \\ &= \int_0^2 x.\frac{1}{2}xdx \\ &= \int_0^2 \frac{x^2}{2}dx \\ &= \frac{4}{3} \end{aligned}$$

Hence, we have, $E(X) = \frac{4}{3}$

Calculating the variance:

$$\begin{aligned} E(X^2) &= \int_{-\infty}^{\infty} x^2 f(x) dx \\ &= \int_0^2 x^2 . \frac{1}{2} x dx \\ &= \int_0^2 \frac{x^3}{2} dx \\ &= \frac{16}{8} \\ &= 2 \end{aligned}$$

Hence, we have, $E(X^2) = 2$

$$\begin{aligned} Var(X) &= E(X - E(X))^2 \\ &= E(X^2) - (E(X))^2 \\ &= 2 - (\frac{4}{3})^2 \\ &= \frac{2}{9} \end{aligned}$$

Hence, we have, $Var(X) = \frac{2}{9}$

Answer:

- $E(X) = \frac{4}{3}$
- $Var(X) = \frac{2}{9}$