

Answer on Question #85465 – Math – Algebra

Question

Use your calculator to find a decimal approximation to three decimal places of both $5/\sqrt{2}$ and $5\sqrt{2}/2$. How similar are they? Why are they either very similar or pretty different?

Solution

Let x_k be a decimal approximation of x to k decimal places. Then:

$$a = \sqrt{2} \approx 1.41421356;$$

$$b = 5/\sqrt{2} \approx 3.53553391;$$

$$c = 5\sqrt{2}/2 = 2.5\sqrt{2} \approx 3.53553391;$$

$$a_3 = 1.414;$$

$$b_3 = 3.536;$$

$$c_3 = 3.536;$$

$$b_3 = c_3.$$

If we make calculations of these numbers with 3 decimal approximations in any step, we lose accuracy and so we will not obtain the main results with the same approximation:

$$a_3 = 1.414;$$

$$b'_3 = 5/a_3 = 5/1.414 \approx 3.53606789 \approx 3.536 = b_3;$$

$$c'_3 = 2.5 * a_3 = 2.5 * 1.414 = 3.535 \neq c_3.$$

We see that after all calculations with 3 decimal places numbers b'_3 and c'_3 are slightly different:

$$b'_3 = 3.536;$$

$$c'_3 = 3.535;$$

$$b'_3 \neq c'_3.$$