Answer on Question #85449 – Math – Statistics and Probability

Question

Let X be a continuous random variable with probability density function given by

$$f(x) = \begin{cases} \frac{1}{2}x\,, & for\ 0 \le x \le 2 \\ 0, & otherwise \end{cases}$$

Determine the mean and variance of this distribution.

Solution

$$\mu = E(X) = \int_{-\infty}^{\infty} x f(x)\,dx = \int_0^2 \frac{1}{2}x^2\,dx = \left[\frac{x^3}{6}\right]_0^2 = \frac{8}{6} - 0 = \frac{4}{3}$$

$$V(X) = \sigma^2 = E((X-\mu)^2) = \int_{-\infty}^{\infty}\left(x-\frac{4}{3}\right)^2 f(x)\,dx = \int_0^2 \frac{1}{2}x\left(x-\frac{4}{3}\right)^2 dx =$$

$$= \frac{1}{2}\int_0^2 \left(x^3 - \frac{8}{3}x^2 + \frac{16}{9}x\right)dx = \frac{1}{2}\left[\frac{x^4}{4} - \frac{8}{9}x^3 + \frac{8}{9}x^2\right]_0^2 =$$

$$= \frac{1}{2}\left(4 - \frac{64}{9} + \frac{32}{9}\right) = \frac{2}{9}$$

Or

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx = \int_0^2 \frac{1}{2}x^3\,dx = \left[\frac{x^4}{8}\right]_0^2 = \frac{16}{8} - 0 = 2$$

$$V(X) = E(X^2) - \left(E(X)\right)^2 = 2 - \left(\frac{4}{3}\right)^2 = \frac{2}{9}.$$

Answer: $\frac{4}{3}$, $\frac{2}{9}$.