Answer on Question #85448 – Math – Statistics and Probability

Question

A continuous random variable X has moment generating function $M(t) = e^{2t^2+3t}$.
Determine the $E(X)$ and $Var(X)$.
Probability that the mean is within two standard deviations.

Solution

$$E(X) = \frac{d}{dt}M(t)|_{t=0}$$
$$M(t) = e^{2t^2+3t}$$
$$\frac{d}{dt}M(t) = \frac{d}{dt}\left(e^{2t^2+3t}\right) = e^{2t^2+3t}(4t+3)$$
$$E(X) = \frac{d}{dt}M(t)|_{t=0} = e^{0+0}(4(0)+3) = 3$$

$$Var(X) = E(X^2) - \left(E(X)\right)^2$$
$$E(X^2) = \frac{d^2}{dt^2}M(t)|_{t=0}$$
$$\frac{d^2}{dt^2}M(t) = \frac{d}{dt}\left(e^{2t^2+3t}(4t+3)\right) = e^{2t^2+3t}(4t+3)^2 + 4e^{2t^2+3t}$$
$$E(X^2) = \frac{d^2}{dt^2}M(t)|_{t=0} = e^{0+0}(4(0)+3)^2 + 4e^{0+0} = 13$$
$$Var(X) = 13 - (3)^2 = 4$$

We have normal distribution, where

$$M(t) = e^{\frac{\sigma^2}{2}t^2+\mu t}$$
$$\mu = 3, \sigma^2 = 4$$

For the normally distributed variable X, 95% of the data is within 2 standard deviations (σ) of the mean (μ):

$$P(\mu - 2\sigma \le X \le \mu + 2\sigma) = 0.9545.$$