

Answer to Question #85442 - Math - Statistics and Probability

Question: A large Computer Company A, subcontract the manufacturing of its circuit board to two Companies , 40% to Company B and 60% to Company C. Company B in turn subcontracts 70% of the orders it receives from Company A to Company D and the remaining 30% to Company E , both subsidiaries of Company B. When the boards are completed by companies D,E and C , they are shipped to Company A to be used in various computer models. It has been found that 1.5% , 1% and 0.5% of the boards from D,E and C respectively prove defection during the 90 day warranty period after a computer is first sold. What is the probability that a given board will be defective during the 90 day period?

Solution: Let us define the following events:

- **B:** The event that the board came from company B
- **C:** The event that the board came from company C
- **D:** The event that the board came from company D
- **E:** The event that the board came from company E

It is clear that, $\mathbf{B} = \mathbf{D} \cup \mathbf{E}$ and the events \mathbf{D} and \mathbf{E} are disjoint. Also the events \mathbf{B} and \mathbf{C} are mutually exclusive and exhaustive.

Given a board, we want to find the probability that it will be defective during the 90 day period. Let \mathbf{X} denote the event that the board will be defective in the warranty period.

$$\begin{aligned} & P(\mathbf{X}) \\ &= P(\mathbf{X} \cap \mathbf{B}) + P(\mathbf{X} \cap \mathbf{C}) \text{ [As, } \mathbf{B} \text{ and } \mathbf{C} \text{ are exclusive and exhaustive]} \\ &= P(\mathbf{X} \cap \mathbf{D}) + P(\mathbf{X} \cap \mathbf{E}) + P(\mathbf{X} \cap \mathbf{C}) \\ &= P(\mathbf{X}|\mathbf{D})P(\mathbf{D}) + P(\mathbf{X}|\mathbf{E})P(\mathbf{E}) + P(\mathbf{X}|\mathbf{C})P(\mathbf{C}) \\ &= P(\mathbf{X}|\mathbf{D})P(\mathbf{D}|\mathbf{B})P(\mathbf{B}) + P(\mathbf{X}|\mathbf{E})P(\mathbf{E}|\mathbf{B})P(\mathbf{B}) + P(\mathbf{X}|\mathbf{C})P(\mathbf{C}) \end{aligned}$$

Observe that, all the values of the above expression has been given in the question. The values are,

$$\begin{aligned} P(\mathbf{X}|\mathbf{D}) &= \frac{1.5}{100} \\ P(\mathbf{X}|\mathbf{E}) &= \frac{1}{100} \\ P(\mathbf{X}|\mathbf{C}) &= \frac{0.5}{100} \\ P(\mathbf{D}|\mathbf{B}) &= \frac{70}{100} \end{aligned}$$

$$P(\mathbf{E}|\mathbf{B}) = \frac{30}{100}$$

$$P(\mathbf{B}) = \frac{40}{100}$$

$$P(\mathbf{C}) = \frac{60}{100}$$

Substituting these values in the expression, we obtain,

$$\begin{aligned} & P(\mathbf{X}) \\ &= \frac{1.5}{100} \frac{70}{100} \frac{40}{100} + \frac{1}{100} \frac{30}{100} \frac{40}{100} + \frac{0.5}{100} \frac{60}{100} \\ &= \frac{4200 + 1200 + 3000}{100^3} \\ &= \frac{8400}{100^3} \\ &= 0.84\% \end{aligned}$$

Answer: Hence, the probability that a given board will be defective within 90 day period is **0.84%**.