Answer on Question #85422 – Math – Calculus

Question

Find, by the first principle, the derivative of $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^3 - 1$ at a point x_0. Hence, find the equations of the tangent and normal to its curve at the point $(-2, -9)$.

Solution

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

$$f(x) = x^3 - 1$$

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h} = \lim_{h \to 0} \frac{(x_0 + h)^3 - 1 - ({x_0}^3 - 1)}{h} =$$

$$= \lim_{h \to 0} \frac{(x_0 + h - x_0)((x_0 + h)^2 + (x_0 + h)x_0 + {x_0}^2)}{h} =$$

$$= \lim_{h \to 0} \frac{h((x_0 + h)^2 + (x_0 + h)x_0 + {x_0}^2)}{h} =$$

$$= \lim_{h \to 0} ((x_0 + h)^2 + (x_0 + h)x_0 + {x_0}^2) =$$

$$= (x_0 + 0)^2 + (x_0 + 0)x_0 + {x_0}^2 = 3{x_0}^2$$

$$f'(x_0) = 3{x_0}^2$$

Find the slope of the tangent to the curve at the point $(-2, -9)$

$$slope_1 = m_1 = f'(-2) = 3(-2)^2 = 12$$

The equation of the tangent to the curve at the point $(-2, -9)$ in point-slope form

$$y - (-9) = 12(x - (-2))$$

The equation of the tangent to the curve at the point $(-2, -9)$ in slope-intercept form

$$y = 12x + 15$$

The equation of the tangent to the curve at the point $(-2, -9)$ in standard form

$$12x - y = -15$$

Find the slope of the normal to the curve at the point $(-2, -9)$

$$slope_2 = m_2 = -\frac{1}{f'(-2)} = -\frac{1}{3(-2)^2} = -\frac{1}{12}$$

The equation of the normal to the curve at the point $(-2, -9)$ in point-slope form

$$y - (-9) = -\frac{1}{12}(x - (-2))$$

The equation of the normal to the curve at the point $(-2, -9)$ in slope-intercept form

$$y = -\frac{1}{12}x - \frac{55}{6}$$

The equation of the normal to the curve at the point $(-2, -9)$ in standard form

$$x + 12y = -110$$