

Answer on Question #85418 – Math – Calculus

Question

Find lower and upper integrals of f , defined on $[-1, 1]$, by $f(x) = \begin{cases} 1, & \text{if } x \text{ is rational} \\ 2, & \text{if } x \text{ is irrational} \end{cases}$
Hence check the integrability of f on $[-1, 1]$.

Solution

Lower integral is lower value multiplied by the length of the segment. It means that the lower integral (LI) is

$$LI = 1 * 2 = 2.$$

Similarly, the upper integral (UI) is

$$UI = 2 * 2 = 4.$$

The function can be considered integrable if for very small delta LI and UI tend to each other. This condition is not satisfied here, so the function is not integrable.