Answer on Question #85291 – Math – Calculus

Question

Evaluate the following integral

$$\int \frac{x\,dx}{(x+2)\sqrt{x+1}}.\qquad\qquad(1)$$

Solution

In the integral (1) we make such a substitution:

$$\sqrt{x+1}=t,\ x+1=t^2,\ x=t^2-1,\ dx=2t\,dt.\qquad\qquad(2)$$

Then

$$\int \frac{x\,dx}{(x+2)\sqrt{x+1}} = \int \frac{(t^2-1)2t\,dt}{(t^2+1)t} = 2\int \frac{(t^2-1)\,dt}{t^2+1} = 2\int \frac{(t^2+1-2)\,dt}{t^2+1} = 2\int \frac{(t^2+1)\,dt}{t^2+1} - 2\int \frac{2\,dt}{t^2+1} =$$

$$=2\int 1\,dt\text{-}4\arctan t + C = 2t - 4\arctan t + C.$$

Going back to substitution (2) we get

$$\int \frac{x\,dx}{(x+2)\sqrt{x+1}} = 2\sqrt{x+1} - 4\arctan\sqrt{x+1} + C = 2(\sqrt{x+1} - 2\arctan\sqrt{x+1}) + C$$

Answer: $\int \frac{x\,dx}{(x+2)\sqrt{x+1}} = 2(\sqrt{x+1} - 2\arctan\sqrt{x+1}) + C$.