Answer to Question #85289 – Math – Calculus

Question

A 24 feet ladder rests against a wall. Let θ be the angle between the top of the ladder and the wall and let x be the distance from the bottom of the ladder to the wall. If the bottom of the ladder slides away from the wall, how fast does x change with respect to θ when θ = pi/3.

Solution

Given that x is the distance from the bottom of the ladder to the wall.

Let the height of the top of the ladder from the base be y.

Then the equation is:

$x^2 + y^2 = 24^2$

$x^2 + y^2 = 576$

$x^2 = 576 - y^2$

Differentiating the above equation with respect to θ we get

$2x\frac{dx}{d\theta} = 0 - 2y\frac{dy}{d\theta}$

$x\frac{dx}{d\theta} = -y\frac{dy}{d\theta}$

$\frac{dx}{d\theta} = \left(-\frac{y}{x}\right)\frac{dy}{d\theta}$

Again we have,

$\tan\theta = \frac{x}{y}$

$\frac{y}{x} = \cot\theta$

At $\theta = \frac{\pi}{3}$, we have,

$$\frac{y}{x} = \cot\frac{\pi}{3} = \frac{1}{\sqrt{3}}$$

$$y = \frac{x}{\sqrt{3}}$$

$$y^2 = \frac{x^2}{3}$$

Again,

$$x^2 + y^2 = 576$$

$$x^2 + \frac{x^2}{3} = 576$$

$$\frac{4x^2}{3} = 576$$

$$x^2 = 144 \times 3$$

$$x = 12\sqrt{3}$$

$$\frac{dy}{d\theta} = \frac{1}{\sqrt{3}}\left(\frac{dx}{d\theta}\right)$$

Then we get,

$$\frac{dx}{d\theta} = \left(-\frac{1}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}\frac{dx}{d\theta}\right)$$

$$\frac{dx}{d\theta} + \frac{1}{3}\frac{dx}{d\theta} = 0$$

$$\frac{4}{3}\frac{dx}{d\theta} = 0$$

$$\frac{dx}{d\theta} = 0$$