Answer on Question #85258 – Math – Algebra

Question

Prove that $2^n > 1 + n^{(1/2^n - 1)}$ for all $n > 2$, using the inequalities.

Solution

For all $n > 0$:

$2^n > 2^0$;

$2^n > 1$;

$1 < 2^n$;

$1/2^n < 1$;

$1/2^n - 1 < 1 - 1$;

$1/2^n - 1 < 0. \quad (1)$

From the inequality (1) for all $n > 1$:

$n^{(1/2^n - 1)} < n^0$;

$n^{(1/2^n - 1)} < 1$;

$1 + n^{(1/2^n - 1)} < 1 + 1$;

$1 + n^{(1/2^n - 1)} < 2^1$;

$1 + n^{(1/2^n - 1)} < 2^n$;

$2^n > 1 + n^{(1/2^n - 1)}. \quad (2)$

The inequality (2) is true for all $n > 1$, so it is true for all $n > 2$. Hence proved.