Answer on Question #85255 – Math – Calculus

Question

If $y = \arcsin(x)$ prove that

$$y_{n+2}(0) = n^2 y_n(0)$$

y_n means nth derivative of n.

Solution

Let $\mathrm{y} = \arcsin(x)$.

The first derivative of $\arcsin(x)$ is given by

$$y' = (\arcsin(x))' = \frac{1}{\sqrt{1-x^2}}$$

This expression can be written as

$$y'\sqrt{1-x^2} = 1$$

If y_n means nth derivative of n, then squaring both sides we get

$$y_1{}^2(1-x^2) = 1$$

Differentiating we get

$$2y_1 y_2(1-x^2) - 2xy_1{}^2 = 0$$

$$y_2(1-x^2) - xy_1 = 0$$

Using the Leibniz rule, we find

$$y_{n+2}(1-x^2) + \binom{n}{1}(-2x)y_{n+1} + \binom{n}{2}(-2)y_n - xy_{n+1} - \binom{n}{1}(1)y_n = 0$$

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - \left(\frac{n(n-1)}{2}(2) + n\right)y_n = 0$$

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$$

When $x = 0$,

$$y_{n+2}(0) - n^2 y_n(0) = 0$$

$$y_{n+2}(0) = n^2 y_n(0)$$