Answer on Question #85224 – Math – Linear Algebra

Question

Q2. If $A = \begin{pmatrix} 1 & 2 & 6 \\ 4 & 11 & 7 \\ 9 & 13 & 3 \end{pmatrix}$

(a) Find the minors of 1,2 and 6.

(b) Find the cofactors of 1,2 and 6.

(c) Evaluate |A|.

(d) A^{-1}

Solution

$$A = \begin{pmatrix} 1 & 2 & 6 \\ 4 & 11 & 7 \\ 9 & 13 & 3 \end{pmatrix}$$

(a) Find the minors of 1,2 and 6.

Minor of 1 is $M_{11} = \begin{vmatrix} 11 & 7 \\ 13 & 3 \end{vmatrix} = 11 \cdot 3 - 7 \cdot 13 = -58$

Minor of 2 is $M_{12} = \begin{vmatrix} 4 & 7 \\ 9 & 3 \end{vmatrix} = 4 \cdot 3 - 7 \cdot 9 = -51$

Minor of 6 is $M_{13} = \begin{vmatrix} 4 & 11 \\ 9 & 13 \end{vmatrix} = 4 \cdot 13 - 11 \cdot 9 = -47$

(b) Find the cofactors of 1,2 and 6.

Cofactor of $a_{ij} = (-1)^{i+j} M_{ij}$

$$C_{11} = (-1)^{1+1} M_{11} = M_{11} = -58$$

$$C_{12} = (-1)^{1+2} M_{12} = -M_{12} = 51$$

$$C_{13} = (-1)^{1+3} M_{13} = M_{13} = -47$$

(c) Evaluate |A|.

$$|A| = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$$

$$|A| = 1 \cdot (-58) + 2 \cdot 51 + 6 \cdot (-47) = -238$$

(d) A^{-1}

We can find inverse matrix by using formula

$$A^{-1} = \frac{1}{|A|} C^T$$

where C is a cofactor matrix

$$C = \begin{pmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{pmatrix}$$

Find cofactors of all the elements

$$C_{21} = (-1)^{2+1} \begin{vmatrix} 2 & 6 \\ 13 & 3 \end{vmatrix} = -(2 \cdot 3 - 6 \cdot 13) = 72$$

$$C_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 6 \\ 9 & 3 \end{vmatrix} = (1 \cdot 3 - 6 \cdot 9) = -51$$

$$C_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 2 \\ 9 & 13 \end{vmatrix} = -(13 - 2 \cdot 9) = 5$$

$$C_{31} = (-1)^{3+1} \begin{vmatrix} 2 & 6 \\ 11 & 7 \end{vmatrix} = (2 \cdot 7 - 6 \cdot 11) = -52$$

$$C_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 6 \\ 4 & 7 \end{vmatrix} = -(7 - 6 \cdot 4) = 17$$

$$C_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 2 \\ 4 & 11 \end{vmatrix} = (11 - 2 \cdot 4) = 3$$

Construct Cofactor Matrix

$$C = \begin{pmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{pmatrix} = \begin{pmatrix} -58 & 51 & -47 \\ 72 & -51 & 5 \\ -52 & 17 & 3 \end{pmatrix}$$

Transpose of the cofactor matrix (adjugate matrix)

$$C^T = \begin{pmatrix} -58 & 72 & -52 \\ 51 & -51 & 17 \\ -47 & 5 & 3 \end{pmatrix}$$

Thus $A^{-1} = \frac{-1}{238} \begin{pmatrix} -58 & 72 & -52 \\ 51 & -51 & 17 \\ -47 & 5 & 3 \end{pmatrix} = \begin{pmatrix} 58/238 & -72/238 & 52/238 \\ -51/238 & 51/238 & -17/238 \\ 47/238 & -5/238 & -3/238 \end{pmatrix}$

Answer:

(a) The minors of 1,2 and 6 are $M_{11} = -58$, $M_{12} = -51$, $M_{13} = -47$

(b) The cofactors of 1,2 and 6 are $C_{11} = -58$, $C_{12} = 51$, $C_{13} = -47$

(c) $|A| = -238$

(d) $A^{-1} = \begin{pmatrix} 58/238 & -72/238 & 52/238 \\ -51/238 & 51/238 & -17/238 \\ 47/238 & -5/238 & -3/238 \end{pmatrix}$.