Answer on Question #84767 – Math – Statistics and Probability

Question

Say true or false with reason. A maximum likelihood estimator is always unbiased.

Solution

False. Consider a sample of $N(\sqrt{\theta}, 1)$. Find the maximum likelihood estimator of θ.

$$L(\vec{x}, \theta) = \frac{1}{(\sqrt{2\pi})^n} \prod_{i=1}^{n} \exp\left(-\frac{(x_i - \sqrt{\theta})^2}{2}\right)$$

$$\log L(\vec{x}, \theta) = -\frac{n}{2}\log(2\pi) - \frac{\sum_{i=1}^{n}(x_i - \sqrt{\theta})^2}{2}$$

$$\frac{\partial}{\partial\theta}\log L(\vec{x}, \theta) = -\frac{1}{2\sqrt{\theta}}\left(n\sqrt{\theta} - \sum_{i=1}^{n} x_i\right)$$

The maximum is for

$\hat{\theta} = \left(\frac{\sum_{i=1}^{n} x_i}{n}\right)^2$ - this is the maximum likelihood estimate for θ.

Find

$$E\hat{\theta} = \frac{E\left(\sum_{i=1}^{n} x_i\right)^2}{n^2} = \frac{\sum_{i=1}^{n} E{x_i}^2 + \sum_{i\neq j} Ex_i Ex_j}{n^2} = \frac{n(\theta+1) + n(n-1)\theta}{n^2} =$$
$$= \theta + \frac{1}{n} \neq \theta$$

This estimate is not unbiased.