Answer on Question #84749 – Math – Calculus

Question

If $f(x)=\dfrac{4x^2-7x-2}{x-2}$, $x\neq 2$ find a $\delta>0$ such that $|f(x)-9|<\dfrac{1}{100}$ for $|x-2|<\delta$.

Hence show that $\lim_{x\to 2} f(x)=9$.

Solution

We want to find a number $\delta>0$ such that

$$\text{if } |x-2|<\delta\,, x\neq 2 \text{ then } |f(x)-9|<\varepsilon$$

But $|f(x)-9|=\left|\dfrac{4x^2-7x-2}{x-2}-9\right|=\left|\dfrac{(4x+1)(x-2)}{x-2}-9\right|=$

$=|4x+1-9|=4|x-2|$

Therefore, we want

$$\text{if } 0<|x-2|<\delta \text{ then } 4|x-2|<\varepsilon$$

that is

$$\text{if } 0<|x-2|<\delta \text{ then } |f(x)-9|<\varepsilon$$

This suggests that we should choose

$$\delta=\frac{\varepsilon}{4}$$

If $\varepsilon=\dfrac{1}{100}$ then $\delta=\dfrac{\varepsilon}{4}=\dfrac{1}{400}$.

Given $\varepsilon>0$, choose $\delta=\varepsilon/4$. If $0<|x-2|<\delta$, then

$|f(x)-9|=\left|\dfrac{4x^2-7x-2}{x-2}-9\right|=\left|\dfrac{(4x+1)(x-2)}{x-2}-9\right|=$

$=|4x+1-9|=4|x-2|<4\delta=4\left(\dfrac{\varepsilon}{4}\right)=\varepsilon$

Therefore, by the definition of limit

$$\lim_{x\to 2} f(x)=9\,, x\neq 2$$