Answer on Question #84747 – Math – Calculus

Question

For which values of k, is the function f, defined as below, continuous at $x = 2$?

$$f(x) = \begin{cases} 3 - kx, & 1 \le x < 2 \\ \dfrac{x^2}{4} - 3, & x \ge 2 \end{cases}$$

Further, at which other points in $[1, \infty)$ is f continuous, and why?

Solution

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (3 - kx) = 3 - 2k$$

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} \left(\frac{x^2}{4} - 3 \right) = \frac{(2)^2}{4} - 3 = -2$$

$$f(2) = \frac{(2)^2}{4} - 3 = -2$$

If the function f is continuous at $x = 2$, then

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = \lim_{x \to 2} f(x)$ and $\lim_{x \to 2} f(x) = f(2)$

Then

$$3 - 2k = -2 \Rightarrow 5 = 2k \Rightarrow k = \frac{5}{2}$$

$$\lim_{x \to 2^-} f(x) = -2 = \lim_{x \to 2^+} f(x) \Rightarrow \lim_{x \to 2} f(x) = -2 \Rightarrow \lim_{x \to 2} f(x) = f(2) \Rightarrow$$

$$\Rightarrow f(x) = \begin{cases} 3 - \dfrac{5}{2}x, & 1 \le x < 2 \\ \dfrac{x^2}{4} - 3, & x \ge 2 \end{cases} \text{ is continuous at } x = 2.$$

The function f is continuous from the left at $x = 1$ as polynomial.
The function f is continuous on $(1, 2)$ as polynomial.
The function f is continuous at $x = 2$.
The function f is continuous on $(2, \infty)$ as polynomial.

The function f is continuous from the left at $x = 1$.
The function f is continuous on $(1, \infty)$.